Math 3

Polynomials Study Guide

Unit 4

Write each polynomial function in standard form. Then determine the end behavior of each.

1. $n = 4m^2 - m + 7m^4$

$f(n) = 7m^4 + 4m^2 - m$

$x \to -\infty, f(x) \to \infty$

$x \to \infty, f(x) \to \infty$

2. $f(t) = 4t + 3t^3 + 2t - 7$

$f(t) = 3t^3 + 6t - 7$

$x \to -\infty, f(x) \to -\infty$

$x \to \infty, f(x) \to \infty$

3. $f(r) = 5r + 7 + 2r^2$

$f(r) = 2r^2 + 5r + 7$

$x \to -\infty, f(x) \to \infty$

$x \to \infty, f(x) \to \infty$

Divide using long division for #s 4 and 5:

4. $(x^3 + 3x^2 - x - 3) \div (x - 1)$

$x^2 + 4x + 3$

5. $(2x^3 - 6x^2 + 4x + 1) \div (x^2 + 3)$

$2x - 6 + \frac{-2x + 19}{x^2 + 3}$

Divide using synthetic division for #s 6 and 7:

6. $(2x^3 - 3x^2 - 18x - 8) \div (x - 4)$

$2x^2 + 5x + 2$ R $-$

7. $(6x^3 - x^2 + 8) \div (x + 2)$

$6x^2 - 13x + 26 + \frac{-44}{x+2}$

Find all factors and solutions of each equation. Sketch a graph and state the end behavior.

8. $f(x) = x^4 + 14x^2 - 32$

$(x^4 + 16x^2) - (2x^2 - 32)$

$x^2(x^2 + 16) - 2(x^2 + 16)$

$(x^2 - 2)(x + 16)$

$(x - \sqrt{2})(x + \sqrt{2})(x + 4i)(x - 4i)$

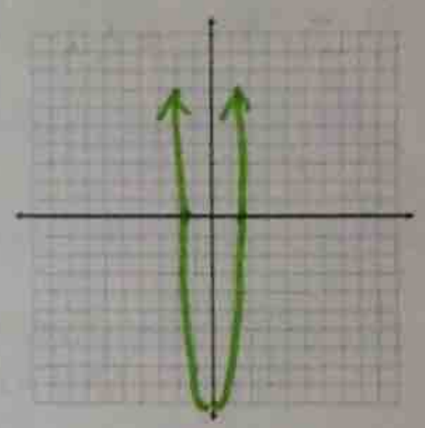
Factors: $(x - \sqrt{2})$ $(x + \sqrt{2})$ $(x + 4i)$ $(x - 4i)$

Roots: $\{\pm\sqrt{2}, \pm 4i\}$

$x \to -\infty, f(x) \to \infty$

$x \to \infty, f(x) \to \infty$

Graph:



9. $f(x) = x^3 - 6x^2 + 8x$

$(x^3 - 4x^2) - (2x^2 + 8x)$

$x^2(x - 4) - 2x(x - 4)$

$(x^2 - 2x)(x - 4)$

$x(x - 2)(x - 4)$

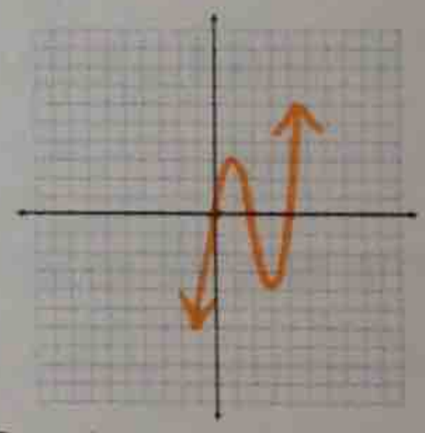
Factors: x $(x - 2)$ $(x - 4)$

Roots: $\{0, 2, 4\}$

$x \to -\infty, f(x) \to -\infty$

$x \to \infty, f(x) \to \infty$

Graph:



10. $f(x) = 6x^3 - 2x^2 - 4x$

$(6x^3 - 6x^2) + (4x^2 - 4x)$

$6x^2(x - 1) + 4x(x - 1)$

$(6x^2 + 4x)(x - 1)$

$2x(3x + 2)(x - 1)$

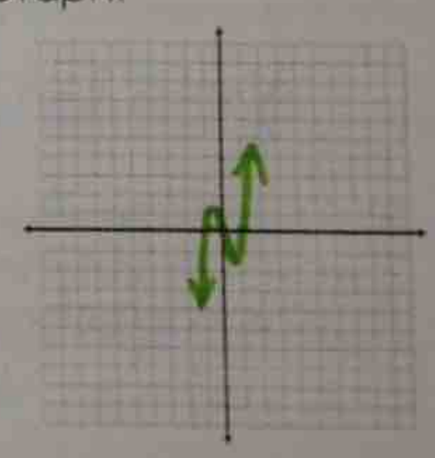
Factors: $2x$ $(3x + 2)$ $(x - 1)$

Roots: $\{0, -\frac{2}{3}, 1\}$

$x \to -\infty, f(x) \to -\infty$

$x \to \infty, f(x) \to \infty$

Graph:



12. $f(x) = x^3 - 3x^2 + 4x - 12$

$(x^3 - 3x^2) + (4x - 12)$

$x^2(x - 3) + 4(x - 3)$

$(x^2 + 4)(x - 3)$

$(x + 2i)(x - 2i)(x - 3)$

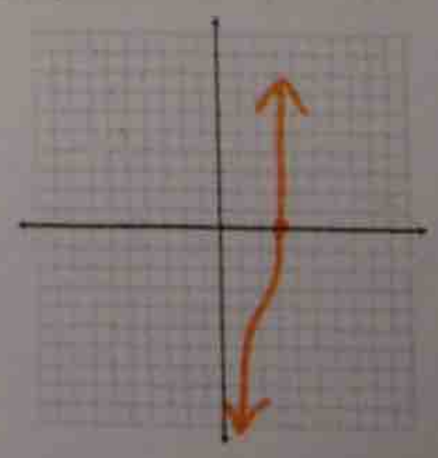
Factors: $(x - 3)$ $(x - 2i)$ $(x + 2i)$

Roots: $\{3, \pm 2i\}$

$x \to -\infty f(x) \to -\infty$

$x \to \infty f(x) \to \infty$

Graph:



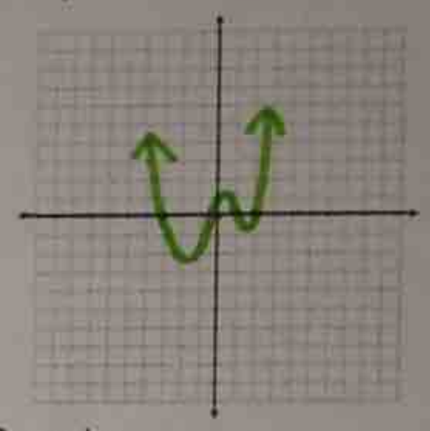
Use the given factor to factor and find all roots of the given polynomial. Sketch a graph and state the end behavior.

13. $f(x) = 3x^4 - x^3 - 22x^2 + 24x$ Factor: (x – 2) Roots: Graph:

$x \to -\infty, f(x) \to \infty$
$x \to \infty, f(x) \to \infty$

x
$(3x - 4)$
$(x + 3)$

$\{-3, 0, 4/3, 2\}$

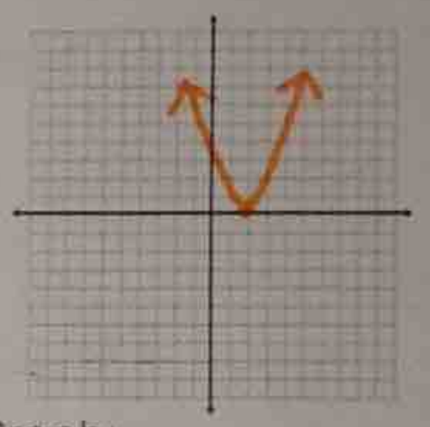


14. $f(x) = x^4 - 4x^3 + 13x^2 - 36x + 36$ Factor: (x – 2) Roots: Graph:

$x \to -\infty, f(x) \to \infty$
$x \to \infty, f(x) \to \infty$

$(x - 2)$
$(x + 3i)$
$(x - 3i)$

$\{2, \pm 3i\}$
↓
M: 2

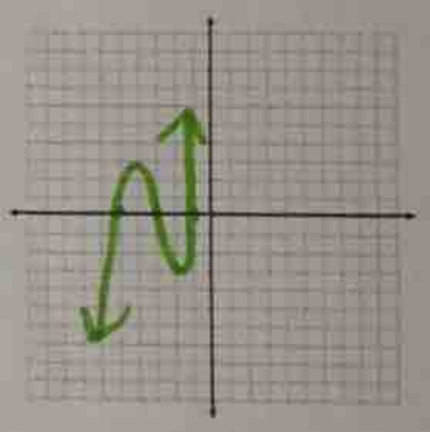


15. $f(x) = x^3 + 9x^2 + 23x + 15$ Factor: (x + 5) Roots: Graph:

$x \to -\infty, f(x) \to -\infty$
$x \to \infty, f(x) \to \infty$

$(x + 3)$
$(x + 1)$

$\{-5, -3, -1\}$



16. When you divide $P(x) = x^3 + 4x^2 - 2x + k$ by $(x - 2)$, the remainder is 4. What is the remainder of P(x) when you divide by $(x + 2)$?

$K = -16$ Remainder $= -4$

17. Write the equation of a polynomial function that has zeros at –3 and $2 \pm i$.

$f(x) = x^3 - 1x^2 - 7x + 15$

18. Write the equation of a polynomial function that has zeros at ¼ and ±5i.

$f(x) = 4x^3 - 1x^2 + 100x - 25$

19. The volume of a box is $x^3 + 4x^2 + 4x$. Explain how you know the box is <u>not</u> a cube.

$V = x(x+2)(x+2)$ In a cube, all the sides are the same length. One of these sides (x) doesn't match the others (x+2)

20. You are drawing a rectangle with side lengths of (3 – x) inches and (x – 7) inches. What is the maximum area that can be obtained from this drawing? What is the value of x that will maximize this drawing? What is the reasonable domain for the area of this rectangle?

$x = 5$
max area $= 4\ in^2$
Reasonable Domain $= (3, 7)$

Expand and simplify each binomial.

21. $(x - 1)^3$ 22. $(3x + 2)^4$ 23. $(4x + 10)^3$

$x^3 - 3x^2 + 3x - 1$

$81x^4 + 216x^3 + 216x^2 + 96x + 16$

$64x^3 + 480x^2 + 1200x + 1000$

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4. $x-1 \overline{)\, x^3+3x^2-x-3}$

x^2+4x+3

$-x^3+1x^2$ ↓

$4x^2-x$

$-4x^2+4x$ ↓

$3x-3$

$-3x+3$

0

5. $x^2+0x+3 \overline{)\, 2x^3-6x^2+4x+1}$

$2x-6$

$-2x^3+0x^2-6x$

$-6x^2-2x+1$

$+6x^2+0x+18$

$-2x+19$

6. $4 \mid 2 \;\; -3 \;\; -18 \;\; -8$

↓ $8 \;\; 20 \;\; 8$

$2 \;\; 5 \;\; 2 \;\; \boxed{0}$

7. $-2 \mid 6 \;\; -1 \;\; 0 \;\; 8$

↓ $-12 \;\; 26 \;\; -52$

$6 \;\; -13 \;\; 26 \;\; \boxed{-44}$

13. $2 \mid 3 \;\; -1 \;\; -22 \;\; +24$

↓ $6 \;\; 10 \;\; -24$

$3 \;\; 5 \;\; -12 \;\; \boxed{0}$

$3x^3+5x^2-12x$

$(3x^3+9x^2)(-4x^2-12x)$

$3x^2(x+3)-4x(x+3)$

$(3x^2-4x)(x+3)$

$x(3x-4)(x+3)$

14. $2 \mid 1 \;\; -4 \;\; 13 \;\; -30 \;\; 36$

↓ $2 \;\; -4 \;\; 18 \;\; -36$

$1 \;\; -2 \;\; 9 \;\; -18 \;\; \boxed{0}$

$(x^3-2x^2)(+9x-18)$

$x^2(x-2)\,9(x-2)$

$(x^2+9)(x-2)$

$(x+3i)(x-3i)(x-2)$

15. $-5 \mid 1 \;\; 9 \;\; 23 \;\; 15$

↓ $-5 \;\; -20 \;\; -15$

$1 \;\; 4 \;\; 3 \;\; \boxed{0}$

x^2+4x+3

$(x+3)(x+1)$

16. $2 \mid 1 \;\; 4 \;\; -2 \;\; K$

↓ $2 \;\; 12 \;\; 20$

$1 \;\; 6 \;\; 10 \;\; 4$

$K+20=4$

$K=-16$

$-2 \mid 1 \;\; 4 \;\; -2 \;\; -16$

↓ $-2 \;\; -4 \;\; 12$

$1 \;\; 2 \;\; -6 \;\; -4$

$R=-4$

17. $x=-3$ $x=2+i$ $x=2-i$

$x+3=0$ $x-2-i=0$ $x-2+i=0$

$(x+3)(x-2-i)(x-2+i)$

$(x+3)(x^2-4x+5)$

$x^3+3x^2-4x^2-12x+5x+15$

$x^3-1x^2-7x+15$

	x	-2	$-i$
x	x^2	$-2x$	$-xi$
-2	$-2x$	$+4$	$+2i$
$+i$	xi	$-2i$	$+1$

18. $x=\frac{1}{4}$ $x=5i$ $x=-5i$

$4x-1=0$ $x-5i=0$ $x+5i=0$

$(4x-1)(x-5i)(x+5i)$

$(4x-1)(x^2+25)$

$4x^3-1x^2+100x-25$

	x	$-5i$
x	x^2	$-5xi$
$+5i$	$+5xi$	$+25$

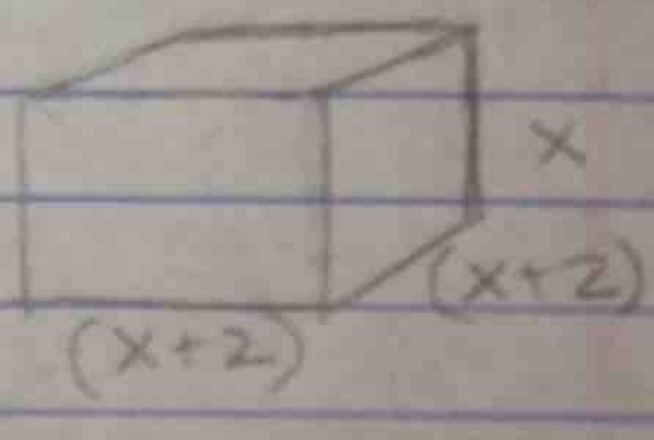
19. $V=x^3+4x^2+4x$

$V=(x^3+2x^2)(2x^2+4x)$

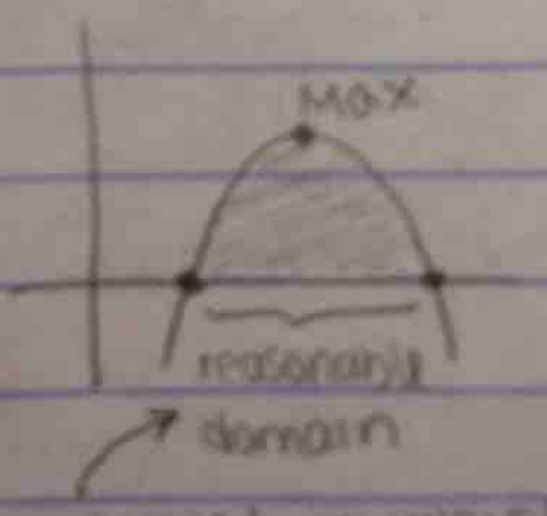
$V=x^2(x+2)2x(x+2)$

$V=(x^2+2x)(x+2)$

$V=x(x+2)(x+2)$



20. Area $=(3-x)(x-7)$



$x=5$

max area $=4\text{ in}^2$

R.D: $(3,7)$

The interval in which x makes the area positive.

21. $(x-1)^3 = (x-1)(x-1)(x-1)$

$1x^3(-1)^0 + 3x^2(-1)^1 + 3x(-1)^2 + 1x^0(-1)^3$

$1x^3 + 3x^2(-1) + 3x(1) + 1(-1)$

$x^3 - 3x^2 + 3x - 1$

22. $(3x+2)^4 = (3x+2)(3x+2)(3x+2)(3x+2)$

$1(3x)^4(2)^0 + 4(3x)^3(2)^1 + 6(3x)^2(2)^2 + 4(3x)^1(2)^3 + 1(3x)^0(2)^4$

$1(81x^4) + 4(27x^3)(2) + 6(9x^2)(4) + 4(3x)(8) + 1(16)$

$81x^4 + 216x^3 + 216x^2 + 96x + 16$

23. $(4x+10)^3 = (4x+10)(4x+10)(4x+10)$

$1(4x)^3(10)^0 + 3(4x)^2(10)^1 + 3(4x)^1(10)^2 + 1(4x)^0(10)^3$

$1(64x^3) + 3(16x^2)(10) + 3(4x)(100) + 1(1000)$

$64x^3 + 480x^2 + 1200x + 1000$